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Optimal Design of Gas Transmission Networks

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ABSTRACT

This study presents a computer algorithm to optimize the design of a gas transmission network. The technique simultaneously determines (1) the number of compressor stations, (2) the diameter and length of pipeline segments, and (3) the operating conditions of each compressor station so that the capital and operating costs are minimized, or profit is maximized. The literature has not reported the solution of such an open-ended problem, although lesser problems have been solved to determine the operating conditions of the gas network for a given configuration. Two solution techniques were used. One was the generalized reduced gradient method, a nonlinear programming algorithm that could be used directly in instances where the capital costs of the compressors were a function of horsepower output but had zero inital fixed cost. The second method was applied to cases in which the capital costs are comprised of a nonzero initial fixed cost plus some function of horsepower output. Here it was necessary to use a branch-and-bound scheme with the nonlinear programming technique mentioned above.

INTRODUCTION

The design or expansion of a gas pipeline transmission system involves a large capital expenditure as well as continuing operation and maintenance costs. Substantial savings have been reported (Flanigan,[4] Graham *et al.*[8]) by improving the system design for a given delivery rate. Both the number and location of compressor stations and the operating parameters of each must be determined to obtain the minimum cost configuration. Such a problem involves both integer and continuous variables because the optimal number of compressor stations is unknown at the outset.

Recent developments in nonlinear programming (optimization) algorithms have made available new techniques for solving such a free configuration design problem for a gas transmission system. This paper describes the gas pipeline, its mathematical formulation (a mixed-integer programming problem), the derivation of various cost functions and constraints, and two techniques for solving the minimum-cost design problem. Two example networks were solved. The first network had gas entry at one point, with delivery to two points. This problem was solved with and without an initial fixed charge for the compressors. The second network was more general, consisting of a multiple entry, multiple delivery network. It was solved for the case of a zero fixed initial charge for the compressor. The procedure should aid in the planning and design of gas pipelines, acquisition of construction sites, and justification of system modification.

THE PIPELINE DESIGN PROBLEM

Suppose a gas pipeline is to be designed to transport a specified quantity of gas per time from the gas wellheads to the gas demand points. The initial states (pressure, temperature, and composition) of the gas at the wellheads and the fixed states of the gas at the demand points are both known. The following design variables need to be determined (1) number of compressor stations; (2) lengths of pipeline segments between compressor stations, that is, station locations; (3) diameters of the pipeline segments; and (4) suction and discharge pressure at each compressor station.

Most published investigations of the above problem have focused on design problems that fix some of the above variables (subproblems of the one posed above). One of the first investigations of optimal operating conditions for a straight (unbranched) natural gas pipeline with compressors in series was performed by Larson and Wong.[12] Their solution technique was dynamic programming, and they found the optimal suction and discharge pressures of a fixed number of compressor stations. The length and diameter of the pipeline segments were considered fixed because dynamic programming was unable to accommodate a large number of decision variables, although it readily handled pressure and compression ratio constraints. A comparison of their approach with the algorithm tested in this paper is discussed later.

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Martch and McCall[13] expanded the unbranched pipeline configuration by adding branches to form a network, and they posed the problem as one of capacity expansion rather than initial design. Nevertheless, the transmission network configuration was predetermined because the optimization technique was dynamic programming and only the pressures were optimized. Rothfarb *et al.*[14] considered the case where the network configuration was not fixed. They investigated the optimal selection of the pipeline diameters from a discrete set of seven possible sizes. No compression facilities were optimized in this investigation. Heuristic procedures for reducing the number of possibilities in the optimization algorithm were introduced. Hence, this algorithm did optimize the configuration, requiring selection of both discrete and continuous variables (although all variables were made discrete in this approach).

Programs offered by computer service companies to optimize gas pipeline networks have been described by Cheeseman[2,3] and Graham *et al.* Both programs required postulation of the network, and a large part of the software was oriented toward solving the steady-state flow and pressure distribution for a single-phase gas network, although Graham *et al.* added parallel branches to provide greater capacity. While complete details on the mathematical approaches were not available, it appeared that the univariate search method, in which one variable was optimized at a time based on the partial derivative, was used in both algorithms. The univariate search method is not considered a very powerful optimization method, especially for constrained optimization problems. Compression facilities were added by trial and error in these methods, and hence were not an integrated part of the optimization procedure. One heuristic feature of Cheeseman's program was that the compression ratios giving the minimum energy consumption should be equal for each station; however, while this may be true for existing compression facilities, it is not necessarily optimum when compressor investment cost is considered.

A more rigorous approach to the problem of simultaneous optimization of compressor sizes and pipeline diameters in a network has been presented by Flanigan, who used a constrained steepest-descent method. Because the variables were not independent, Flanigan used linearized constraint equations and required that the solution at each step in the optimization procedure represent a feasible point. This could increase the computing time and required selection of dependent and independent variables, necessitating "judgment and experience" according to Flanigan. This algorithm did not consider the optimization of the number of compressors to be used in the network, nor did it explicitly treat inequality constraints. Another constrained optimization procedure, based on Kuhn-Tucker conditions, was proposed by Hax,[9] who used it to determine optimum operating conditions; this method was much more limited than Flanigan's.

PROBLEM FORMULATION

Fig. 1 illustrates a simplified network used as an example of the problem definition and the solution technique. The configuration of the pipeline and the characteristics of the numbering system for the compressor station and pipeline segments are shown. Each compressor station is represented by a node and each pipeline segment by an arc between two nodes. Pressure increases at a compressor station and decreases along a pipeline segment. The transmission system is horizontal. Although a simple example was selected to illustrate the transmission system, a much more complicated network can be accommodated, including various branches and loops, at the expense of increased computer solution time.

Fig. 1 shows these elements.

n_c = total compressors;

$n_c - 1$ = suction pressures (the initial entering pressure is known);

n_c = discharge pressures;

$n_s = n_c + 1$ = pipeline segment lengths (note there are two segments issuing at the branch point);

and

$n_s = n_c + 1$ = pipeline segment diameters.

Each pipeline segment is associated with five variables: (1) the flow rate, q; (2) the inlet pressure, p_d (discharge pressure from upstream compressor); (3) the outlet pressure, p_s (suction pressure of downstream compressor); (4) the pipeline segment diameter, d; and (5) the pipeline segment length, l. Where the mass flow rate through the pipeline is predetermined, each compressor is assumed to lose 0.5 percent of the gas transmitted. In this case, only the last four variables of each pipeline segment need be determined.

The objective function of the pipeline is posed

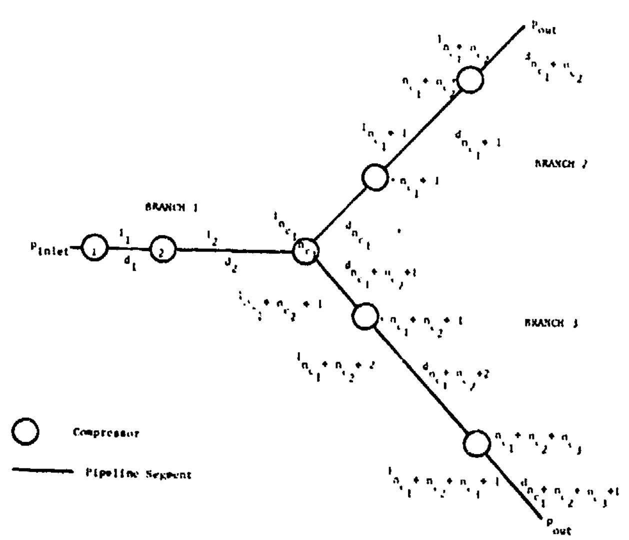

FIG. 1 — PIPELINE CONFIGURATION WITH THREE BRANCHES.

as a minimum cost problem. The objective function is comprised of the yearly operating and maintenance costs of the compressors plus the sum of the discounted capital costs of the pipe and compressors. Each compressor is assumed adiabatic, with an inlet temperature equal to that of the surroundings. An efficiency factor, η_s, can be used to correct for the mechanical efficiency of the compressor (assumed to be 100 percent in this study). One compressor's rate of work can be described as

$$\eta_s W' = 0.08531 \frac{\gamma}{\gamma - 1} T_s \{1 - \left(\frac{p_d}{p_s}\right)^{z(\gamma-1/\gamma)}\}, \quad \ldots \ldots \ldots (1)$$

where W' is expressed in horsepower, γ is the ratio of the specific heats, the suction temperature, T_s, is expressed in °R, and z is the gas compressibility factor.

Operating and maintenance charges per year, O_y, can be related directly to horsepower (Cheeseman) and have been estimated at \$8 to \$14/hp-year (Martch and McCall). The annualized capital costs for each pipeline segment, C_s, depend on the pipe diameter and length, and have been estimated at \$870/in.-mile-year (Martch and McCall). Fig. 2 shows two cost curves for the capital expense of the compressors. Line A indicates cost is a linear function of horsepower (C_c, the compressor capital cost, is equal to \$70/hp-year), passing through the origin. Line B also assumes a linear function of horsepower (C_o is equal to \$69.50/hp-year) with a fixed initial capital outlay of \$10,000, to account for installation, foundation, and other costs. For Line A, the objective function expressed in dollars per year is

$$f = \sum_{i=1}^{n_c} (O_y + C_c)\, 0.08531 \frac{T_s}{\eta_s} \left(\frac{\gamma}{\gamma-1}\right) \{1 - \left(\frac{p_{d_i}}{p_{s_i}}\right)^{z(\gamma-1)/\gamma}\} + \sum_{j=1}^{n_s} C_1 l_j d_j \quad \ldots \ldots (2)$$

Although the objective function costs are linear with respect to compressor output horsepower, the over-all objective function is nonlinear. Thus, any continuous cost function with respect to horsepower can be used for O_y and C_c, and these cost functions do not have to be linear to use the mathematical technique.

For Line A of Fig. 2, where an initial fixed charge does not exist for the compressors, the transmission network problem can be solved solely by a nonlinear programming algorithm. On the other hand, if the capital expense of the compressors has an initial fixed charge (Line B of Fig. 2), then the transmission line problem becomes more difficult and usually must be solved by a branch-and-bound algorithm.

For Line A of Fig. 2, a branch-and-bound technique is not required because of the way the objective function is formulated. If the ratio $p_{d_i}/p_{s_i} = 1$, the term involving Compressor i vanishes from the first summation in the objective function, which is equivalent to deleting Compressor i in a branch-and-bound scheme. The pipeline segments joined at Node i may have different diameters. If Line B represents the compressor costs, the fixed incremental cost for each compressor in the system at zero horsepower (C_{c_o}) would *not* be multiplied by the term in the square brackets of Eq. 2. Instead, C_{c_o} would be added, whether or not Compressor i is in the system, if the nonlinear programming technique was to be used alone. Hence, for Line B of Fig. 2, a different solution procedure, namely, a branch-and-bound one with nonlinear programming, must be used, resulting in much longer computer times.

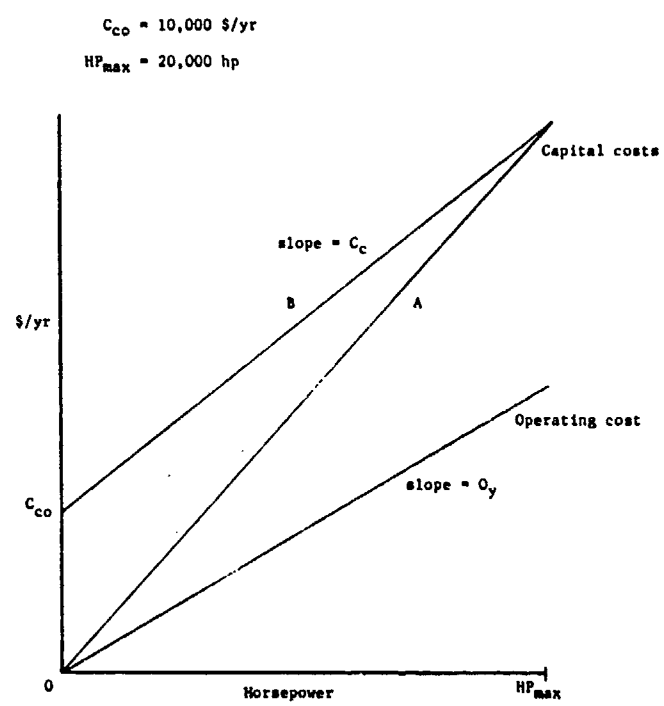

FIG. 2 — CAPITAL AND OPERATING COSTS OF COMPRESSORS.

THE INEQUALITY CONSTRAINTS

Each compressor is constrained so the discharge pressure is greater than or equal to the suction pressure,

$$\frac{p_{d_i}}{p_{s_i}} \geq 1 \qquad i = 1, 2, \ldots, n_c, \quad \ldots \ldots \ldots (3)$$

and the compression ratio does not exceed some prespecified maximum limit K,

$$\frac{p_{d_i}}{p_{s_i}} \le K_i \quad i = 1, \ldots, n_c \quad \ldots\ldots (4)$$

In addition, upper and lower bounds are placed on each variable.

$$(p_{d_i})_{min} \le p_{d_i} \le (p_{d_i})_{max} \quad \ldots\ldots (5a)$$

$$(p_{s_i})_{min} \le p_{s_i} \le (p_{s_i})_{max}, \quad \ldots\ldots (5b)$$

$$(l_i)_{min} \le l_i \le (l_i)_{max}, \quad \ldots\ldots (5c)$$

and

$$(d_i)_{min} \le d_i \le (d_i)_{max}. \quad \ldots\ldots (5d)$$

THE EQUALITY CONSTRAINTS

Two classes of equality constraints exist for the transmission system. First, the length of the system is fixed. There are two length constraints for Fig. 1.

$$\sum_{j=1}^{n_{c_1}} l_j + \sum_{j=n_{c_1}+1}^{n_{ct}} l_j = l_1^*,$$

$$\sum_{j=1}^{n_{c_1}} l_j + \sum_{n_{ct}+1}^{n_c} l_j = l_2^*, \quad \ldots\ldots (6)$$

where n_{c_i} is the number of compressors in Branch i, n_{ct} is the total number of compressors in the first two branches, and l_i^* is the total length between input and a given output. This type of constraint does not reflect accurately the need to select the optimal branch point. That would require altering the distance constraints to account for the geometry of the supply-and-demand points. A simplified constraint form was used in this study; the optimization of the branch location will be pursued later. The flow equation (the Weymouth relation[7]) also must hold in each pipeline segment,

$$q_j = A\, d_j^{8/3} \left[\frac{p_{d_j}^2 - p_{s_j}^2}{l_j} \right]^{1/2}, \quad \ldots\ldots (7)$$

where $A = 8.71 \times 10^8$ and q_j is the flow rate in Segment j. To avoid problems in taking square roots, Eq. 7 is squared to yield

$$A^2 d_j^{16/3} (p_{d_j}^2 - p_{s_j}^2) - l_j q_j^2 = 0 \quad \ldots (8)$$

TECHNIQUES OF SOLUTION

If the capital costs in the problem are described by Line A in Fig. 2, then the problem can be solved directly by a nonlinear programming algorithm. Of the many existing algorithms that might be used,[10] the generalized reduced gradient method[1] has been found to be generally superior to other constrained multivariable methods.

The concept of the reduced gradient can be illustrated with a problem of two variables [$\bar{x} = (x_1, x_2)^T$].

Minimize $f(\bar{x}) \qquad \bar{x} \in E^2$

subject to $h(\bar{x}) = 0 \quad \ldots\ldots (9)$

The total derivatives of the objective function and the equality constraint are

$$df = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 \quad \ldots (10)$$

and

$$dh = \frac{\partial h}{\partial x_1} dx_1 + \frac{\partial h}{\partial x_2} dx_2 \quad \ldots (11)$$

Observe that for feasible differential displacements along the linearized equality constraint, Eq. 11 equals zero. Thus, one can solve for one displacement and eliminate the other from Eq. 10.

$$dx_1 = -\left[\frac{\partial h}{\partial x_2} \Big/ \frac{\partial h}{\partial x_1} \right] dx_2 \quad \ldots (12)$$

and

$$df = \left\{ -\left[\frac{\partial h}{\partial x_2} \Big/ \frac{\partial h}{\partial x_1} \right] \frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2} \right\} dx_2 \quad \ldots\ldots (13)$$

In this example, x_1 is eliminated as an independent variable and the objective function is reduced to an unconstrained function of one independent variable, x_2, and one dependent variable, $x_1 = x_1(x_2)$. Once x_2 is determined by the minimization, x_1 is calculated from the difference equivalent of Eq. 12 for small displacements. Thus, a simple unconstrained minimization along the direction, df/dx_2 (Eq. 13) yields a constrained minimum of $f(\bar{x})$. In Eq. 13, df/dx_2 is known as the reduced gradient because it is expressed in terms of independent variables only. This concept is equivalent to that used by Flanigan.

In vector notation for n_v variables, of which m_v are dependent (subscript D) and $(n_v - m_v)$ are independent (subscript I), and m_v independent equality constraints exist, the equations corresponding to Eqs. 10 through 13 are

$$df(\bar{x}) = \bar{\nabla}^T_{\bar{x}_I} f \cdot d\bar{x}_I + \bar{\nabla}^T_{\bar{x}_D} f \cdot d\bar{x}_D \quad , \qquad (10a)$$

$$d\bar{h}(\bar{x}) = \bar{\nabla}^T_{\bar{x}_I} \bar{h}\; d\bar{x}_I + \bar{\nabla}^T_{\bar{x}_D} \bar{h}\; d\bar{x}_D \quad , \qquad (11a)$$

$$d\bar{x}_D = - \left(\nabla^T_{\bar{x}_D} \bar{h} \right)^{-1} \bar{\nabla}^T_{\bar{x}_I} \bar{h}\; d\bar{x}_I , \qquad (12a)$$

and

$$df(\bar{x}) = \left[\bar{\nabla}^T_{\bar{x}_I} f - \bar{\nabla}_{\bar{x}_D} f \left(\bar{\nabla}^T_{\bar{x}_D} \bar{h} \right)^{-1} \bar{\nabla}^T_{\bar{x}_I} \bar{h} \right] d\bar{x}_I \qquad (13a)$$

Nonlinear equality constraints are transformed into equality constraints by squared slack variables, except for the trivial bounds on the variables.

Let $\bar{g}_r^{(k)}$ be the value of the reduced gradient vector evaluated at some feasible point $\bar{x}^{(k)}$ (defined by Eq. 13a). The generalized reduced gradient method begins the search for the minimum in the direction $\bar{s}^{(0)}$ defined as

$$\bar{s}^{(0)} = -\bar{g}_r^{(0)} .$$

Subsequent search directions are chosen by a conjugate direction method such as the Fletcher-Reeves recursion formula[5] that states

$$\bar{s}^{(k+1)} = -\bar{g}_r^{(k+1)} + \bar{s}^{(k)} \frac{\bar{g}_r^{(k+1)^T} \cdot \bar{g}_r^{(k)}}{\bar{g}_r^{(k)^T} \cdot \bar{g}_r^{(k)}} \qquad (14)$$

It can be shown that these search directions are constrained to the hyperplanes of the locally linearized active constraints.

In the presence of nonlinear constraints, the univariant minimizations often lead to unfeasible $\bar{x}$ vectors. A move into the unfeasible region is limited by heuristic criteria.[1] Feasibility is then regained by using Newton's method to solve the set of nonlinear equations $\bar{h}(\bar{x}_D)$ holding $\bar{x}_I$ constant.

$$\bar{x}'_D = \bar{x}_D - (\bar{\nabla}^T_{\bar{x}_D} \bar{h})^{-1} \bar{h}\; (\bar{x}_D) \qquad (15)$$

where $\bar{x}'_D$ designates a point nearer the feasible region. Iteration by Eq. 15 is continued until the constraints reach the desired tolerance. The active constraints then are relinearized and a new reduced gradient and search direction are calculated. If Eq. 15 does not converge, the variable basis is altered (selected dependent and independent variables are interchanged) and Eq. 15 is reapplied. This linearization with Newton's method, rather than a Hardy-Cross type method, is used to achieve the flow and pressure distribution in the network.

BRANCH-AND-BOUND SOLUTION TECHNIQUE

As explained, with a fixed initial capital investment for the compressors as indicated by Line B in Fig. 2, a nonlinear programming algorithm cannot directly solve the transmission line problem. Instead, a branch-and-bound technique combined with nonlinear programming must be used to handle the integer variable.

A branch-and-bound algorithm is nothing more than an organized enumeration technique, used to delete certain portions of the possible solution set from consideration. A tree is formed of nodes and branches (arcs). Each branch in the tree represents a nonlinear problem without integer variables that is solved as explained above.

For example, in Fig. 3, Node 1 in the tree represents the original problem as posed by Eqs. 2 through 8. When the problem at Node 1 is solved, it provides a lower bound on the solution of the problem posed by the cost function of Line B in Fig. 2. Note that Line A always lies below Line B. (If the problem at Node 1 has no feasible solution, neither does the more complex problem.) With the solution of the problem at Node 1, a decision is made to partition on one of the three integer variables, n_{c_1}, n_{c_2}, or n_{c_3}, which are the number of compressors in Branches 1, 2, and 3, respectively. The partition variable is determined when the smallest average compression ratio for all the branches in the transmission system is calculated by adding all compression ratios in each branch and dividing by the number of compressors. The number of compressors in the

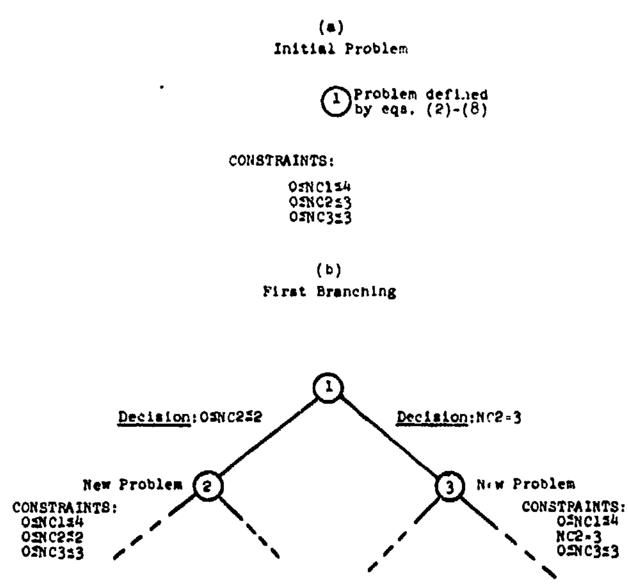

FIG. 3 — PARTIAL TREE AND BRANCHES FOR THE EXAMPLE PROBLEM.

branch with the smallest ratio becomes the partition variable. For example, in Fig. 3 the partition variable was calculated to be n_{c_2}.

After choosing the partition variable, the next step was to determine how the variable should be partitioned. Each compressor in the transmission line branch associated with the partition variable was checked, and if any compressor operated at less than 10-percent capacity, it was assumed to be unnecessary in the line. (If all operated at greater than 10-percent capacity, the compressor with the smallest compression ratio was deleted.) For example, with n_{c_2} selected, and one of three possible compressors at less than 10-percent capacity, the first partition would lead to the tree shown in Fig. 3b; n_{c_2} would be either 3 or $0 \leq n_{c_2} \leq 2$. Thus, at each node in the tree the upper and lower bound on the number of compressors in each branch of the pipeline is readjusted.

The nonlinear problem at Node 2 will be the same as at Node 1, with two exceptions. First, the maximum number of compressors permitted in Branch 2 of the transmission line is now two. Second, the objective function is changed. From the lower bounds, the minimum number of compressors in each branch of the pipeline is known. For the lower bound, the costs related to Line B in Fig. 2 apply; for compressors in addition to the lower bound and up to the upper bound, the costs are represented by Line A.

TABLE 1 — COMPARISON WITH RESULTS OF LARSON AND WONG

Suction Pressure	Larson and Wong (psia)	This Study (psia)
p_{s_1}	500	500.0
p_{s_2}	620	595.7
p_{s_3}	820	763.6
p_{s_4}	580	588.7
p_{s_5}	520	526.1
p_{s_6}	620	628.2
p_{s_7}	750	755.1
p_{s_8}	690	697.5
p_{s_9}	810	811.0
$p_{s_{10}}$	590	596.8
Discharge Pressure	(psia)	(psia)
p_{d_1}	800	800.0
p_{d_2}	1,000	953.1
p_{d_3}	1,000	1,000.0
p_{d_4}	760	765.5
p_{d_5}	840	841.8
p_{d_6}	950	951.5
p_{d_7}	900	900.0
p_{d_8}	1,000	1,000.0
p_{d_9}	1,000	1,000.0
$p_{d_{10}}$	770	767.3
p_{out}	500	500.0
	Objective function $= 1.135242 \times 10^5$	Objective function $= 1.1325169 \times 10^5$

As the decision tree descends, the solution at each node becomes more constrained until Node i is reached, in which the upper and lower bounds for the number of compressors in each pipeline branch are the same. The solution at Node i will be feasible, but not necessarily optimal, for the general problem. Nevertheless, the important point is that the solution at Node i is an upper bound on the solution of the general problem.

As the search continues through the rest of the tree, if the value of the objective function at a node is greater than that of the best feasible solution found so far, then it is not necessary to continue down that branch. The objective function of any subsequent solution found in that branch would be larger than the solution already found. Thus, we can fathom the node, that is, end the search down that branch of the tree. The next step is to backtrack up the tree and continue searching through other branches until all nodes in the tree have been fathomed. Another reason to fathom a particular node is if no feasible solution exists to the nonlinear problem at Node i; then all subsequent nodes below Node i also will be unfeasible.

At the end of the search, the best solution found is the solution to the general problem.

NUMERICAL RESULTS

To test the effectiveness of the proposed solution technique, an example problem formulated by Larson and Wong was solved using as the objective function the total horsepower of the compressors in a long, straight pipeline. In their problem, the length and diameter of each pipeline segment were fixed. Table 1 shows our results compared with those of Larson and Wong, who used dynamic programming. Both the suction and discharge pressures differ from those of Larson and Wong in many instances because their solution did not satisfy the constraints in their problem. Solving the nonlinear problem required 10 seconds of

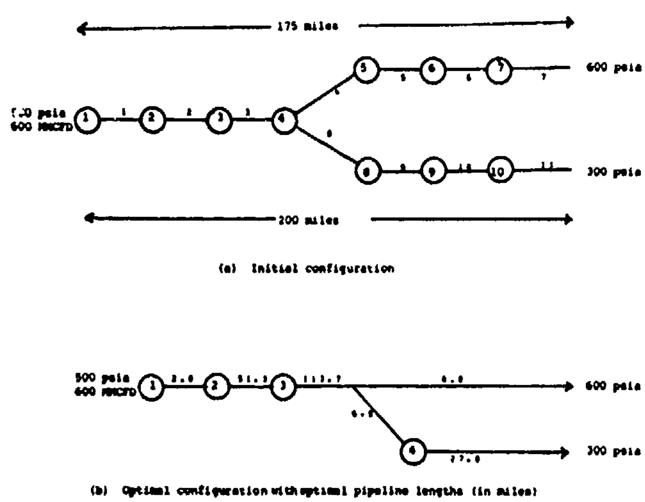

FIG. 4 — INITIAL GAS TRANSMISSION SYSTEM AND FINAL OPTIMAL SYSTEM USING THE COSTS OF LINE A, FIG. 2.

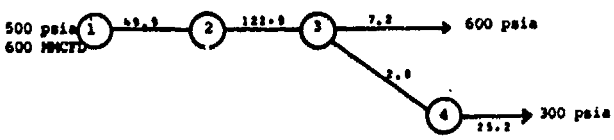

FIG. 5 — OPTIMAL CONFIGURATION USING THE COSTS OF LINE B, FIG. 2.

central processing time on a CDC 6600 computer.

A more complicated network, using the initial configuration shown in Fig. 4a and the cost relation of Line A in Fig. 2, was then optimized. This cost relationship allowed direct application of nonlinear programming, but it did require the initial postulation of compressor locations. The technique, when converged, indicated which compressor stations should be deleted. The maximum number of compressors in Branches 1, 2, and 3 was specified to be 4, 3, and 3, respectively. The entry pressure was 500 psia at a flow rate of 600 MMcf/D, and the two output pressures were set at 600 psia and 300 psia, respectively, for Branches 2 and 3. The total length of Branches 1 and 2 was constrained at 175 miles and of Branches 1 and 3 at 200 miles. While this geometry was unrealistic, it simplified the pipeline length constraints somewhat. The upper bound on the pipeline diameter in Branch 1 was set at 36 in. and in Branches 2 and 3 at 18 in., and the lower bound on the diameters of all pipeline segments at 4 in. These bounds were arbitrary and could be adjusted after the results were obtained. A lower bound of 2 miles was placed on each pipeline segment to assure that the natural gas was at ambient conditions when it entered the next compressor in the pipeline.

Fig. 5 compares the optimal gas transmission network with the original network. From an unfeasible starting configuration with 10-mile pipeline segments, the nonlinear optimization algorithm reduced the objective function from the first feasible state of \$1.399 × 10^7/year to \$7.289 × 10^6/year, a savings of close to \$7 million. Of the 10 possible compressor stations, only four remained in the final optimum network. Table 2 shows the final state of the network. The solution of this problem required 353 seconds of central processing time on a CDC 6600.

The nomenclature in Table 2 indicates that if the suction pressure from the ith pipeline segment was equal to the discharge pressure in the $(i+1)$th segment, no compressor existed (and no cost was added to the objective function, according to Eq. 2). Note that six compressors were removed. Also, the constraints on pipe length and diameter were active in most pipes, indicating different constraint values would give different converged results. Note also that the optimal compression ratio was not the same for all compressors for this problem because of the effect of intervening pipe sections.

The problem described above and represented by Fig. 1 was solved again using the costs represented in Fig. 2 by Line B instead of Line A. Fig. 5 and Table 3 present the results of the computations. Note that Compressor 3 remained in the final configuration but with a compression ratio of 1, indicating it was not performing. This means it was cheaper to have two pipeline segments in Branch 1 and two compressors operating at about one-half capacity, plus a penalty of \$10,000, than to have one pipeline segment and one compressor operating at full capacity. Compressor 3 performing no work represented just a branch in the line plus a cost penalty. About 900 seconds were required on the CDC 6600 to obtain the optimal solution using the branch-and-bound technique.

The final example solved is shown in Fig. 6a, with tabulated results shown in Table 4. This

TABLE 2 — VALUES OF OPERATING VARIABLES FOR THE OPTIMAL NETWORK CONFIGURATION USING THE COSTS OF LINE A, FIG. 2

Pipeline Segment	Discharge Pressure (psia)	Suction Pressure (psia)	Diameter (in.)	Length (mile)	Flow Rate (MMcf/D)	Compressor Station	Compression Ratio
1	119.188	715.399	35.0	2.0	597.0	1	1.44
2	1,000.000	889.352	32.4	51.3	594.0	2	1.40
3	1,000.000	735.786	32.4	113.7	591.0	3	1.12
4	735.786	703.812	18.0	2.0	294.0	4	1.00
5	703.812	670.657	18.0	2.0	292.6	5	1.00
6	670.657	636.133	18.0	2.0	291.1	6	1.00
7	636.133	600.000	18.0	2.0	289.7	7	1.00
8	735.786	703.812	18.0	2.0	294.0	8	1.26
9	885.252	859.128	18.0	2.0	292.6	9	1.00
10	859.128	832.457	18.0	2.0	291.1	10	1.00
11	832.457	300.000	18.0	27.0	289.7		

TABLE 3 — VALUES OF OPERATING VARIABLES FOR THE OPTIMAL NETWORK CONFIGURATION USING THE COSTS OF LINE B, FIG. 2

Pipeline Segment	Discharge Pressure (psia)	Suction Pressure (psia)	Diameter (in.)	Length (mile)	Flow Rate (MMcf/D)	Compressor Station	Compression Ratio
1	954.488	837.246	32.3	49.9	597.0	1	1.91
2	1,000.000	699.734	32.3	122.9	594.0	2	1.19
3	699.734	600.000	15.2	2.2	295.5	3	1.00
4	699.734	665.684	18.0	2.0	295.5	4	1.43
5	952.200	300.000	16.9	25.2	294.0		

TABLE 4 — VALUES OF OPERATING VARIABLES FOR THE OPTIMAL NETWORK CONFIGURATION USING COSTS OF LINE A, FIG. 2

Pipeline Segment	Discharge Pressure (psia)	Suction Pressure (psia)	Diameter (in.)	Length (mile)	Flow Rate (MMcf/D)	Compressor Station	Compression Ratio
1	729.5	724.6	28.53	2.0	398.0	1	1.46
2	1,000.0	921.1	31.75	33.30	594.0	2	1.46
3	980.6	972.2	20.18	2.0	237.7	3	1.38
4	982.6	974.2	20.12	2.0	236.5	4	1.06
5	1,000.0	647.1	31.75	131.70	585.0	5	1.01
6	647.1	641.5	25.37	2.0	291.0	6	1.03
7	684.7	600.0	24.83	27.0	290.0	7	1.01
8	731.7	724.6	20.53	2.0	199.0	8	1.00
9	980.6	973.8	24.34	2.0	351.6	9	1.07
10	981.0	974.2	24.27	2.0	351.6	10	1.00
11	647.1	510.5	14.22	2.0	291.0		
12	510.4	300.0	14.01	2.0	290.0		

multiple input-output example was solved to show how the technique can be applied to more general networks. A bypass network also was added to show the versatility in describing all possible network configurations. The bypass segment required modification of the original problem structure because the flow through the bypass line merged with the regular network.

CONCLUSIONS

We have described a workable procedure for optimal gas transmission line design that can be extended to treat much larger and more complex networks, at the expense of considerable computer time.

NOMENCLATURE

A = constant in Weymouth equation
C_c = annualized capital cost coefficient for compressor
C_1 = annualized capital cost coefficient for pipe
d_j = diameter of jth pipe segment
E = Euclidean space
f = cost (objective) function
$g_r^{(k)}$ = reduced gradient at kth iteration
h = equality constraint vector
K_i = upper bound on compression ratio
l_j = length of jth pipe segment
l_j^* = total length of pipeline between supply and demand points
n_c = total number of compressors in network
n_{c_i} = total number of compressors in Branch i of network
n_{ct} = total number of compressors in a loop composed of two branches
n_s = total number of pipeline segments
n_v = total number of variables
m_v = total number of equality constraints
O_y = yearly operating cost coefficient for compressor
p_{d_i} = discharge pressure of ith compressor
p_{s_i} = suction pressure of ith compressor
q_j = volumetric flow rate in jth pipeline segment
$s^{(k)}$ = search direction at kth iteration in nonlinear programming
T_s = compressor suction temperature
W' = compressor work
x_i = ith optimization variable
z = gas compressibility factor
η_s = compressor efficiency
γ = ratio of specific heats
$\bar{\nabla}$ = gradient operator

SUBSCRIPTS

D = dependent variable
I = independent variable

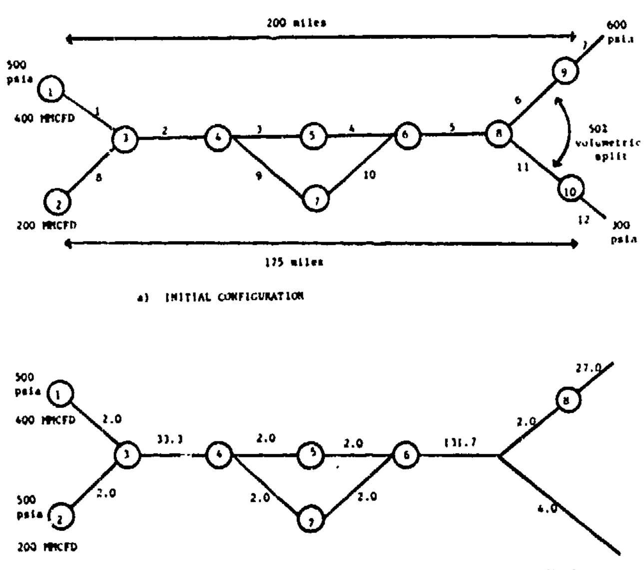

FIG. 6 — INITIAL GAS TRANSMISSION SYSTEM AND FINAL OPTIMAL SYSTEM USING THE COSTS OF LINE A, FIG. 2.

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